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SCHOOL OF PHYSICS AND ASTRONOMY

MPHYS FINAL PROJECT REPORT

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# Comparing Simulations of Pedestrian Motion

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## Abstract

We investigate the difference between cellular automaton based models and social forces model in the simulation of pedestrian dynamics. Recent findings with a cellular automaton of the effect of communication on the evacuation behaviour of pedestrians were reproduced qualitatively. The overall trend of increasing evacuation efficiency was confirmed. The specific result that the optimal fraction of agents communicating is 0.6 could only be reproduced for a specific communication interval of 10 seconds. A second point of comparison was the behaviour of pedestrians at a T-Junction. This has been previously studied using totally asymmetric exclusion process both analytically and using simulations. This scenario was simulated using the social forces model and qualitative agreement was found with the earlier results.

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# 1 Introduction

The repeat occurrences of mass panics that lead to the death of many, such as the 1990 Mecca panic, which killed over 100 people, is a strong motivation for a robust understanding of crowd dynamics. As such simulations cannot be recreated experimentally, a realistic model is necessary to investigate such effects. Numerous different models exist and understanding the limitations and realism of the different existing models is the aim of this project. Specifically, the effect of communication and directing on pedestrian evacuation were investigated in the case of a room and a corridor.

The case for an investigation by physicists is twofold. On one hand, physics has been concerned with particle motion since its inception and all the approaches that exist in traditional physics have been successfully applied to pedestrian motion. Pedestrians have been modelled with Newton equations or stochastically with Langerin and Fokker-Planck equations as well as fluids. On the other hand, this problem seems to be well suited to a reductionist, physicist approach. It is a complex system in which it is possible to reproduce emergent phenomena with a few fundamental rules. Emergent means that the observed phenomena are not build into the system but appear because of the interaction between the simple parts of the system.

Pedestrian dynamics is a complex system involving interactions between individuals. These occur over a range of distances; a pedestrian will interact more strongly with another pedestrian five meters away but on a collision course than a pedestrian half a meter away in parallel motion. Through these interactions arise several self-organization phenomena (such as lane formation) and only a few models exist that reproduce empirically observed behaviour accurately. Strictly speaking the term "pedestrian" refers to a person and "particle" is the representation of a person in a model. An "agent" is a particle in a model that acts autonomously e.g. communicates and makes decisions.

In the remainder of section 1 the empirical observations and approaches of simulating pedestrian dynamics are described. In section 2 the model used in our simulation is detailed and in sections 3 and 4 we present our results. In section 5 we conclude with a summary and an outlook on future work. Sections 1 and 2 closely follow the form of the interim report while material that has already been used in the interim report is clearly labelled in the remaining sections.

## 1.1 Empirical observations of Pedestrian Dynamics

For the motion of pedestrians, relatively little empirical data exists because of the difficulty of analysing experimental data. A common method of data collection is by analysing video footage. For the case of panics and evacuations almost no empirical data is available due to the unexpected nature of such occurrences.

Pedestrians have a desired walking speed that depends on their purpose, external conditions, sex (men are 10 % faster than women) and age (older people walk slower) of the walker. Weidmann [1] compiled average values of pedestrian velocity, shown in Table 1. The velocities in crowds follows a Gaussian distribution with a mean of 1.34 m/s and a standard deviation of 0.26 m/s. Pedestrians can accelerate and decelerate almost instantly and have a typical step length of 0.65 m and step frequency of 2 Hz.

Crowds display a large range of collective phenomena, a short discussion of their behaviour under panic conditions will be given here based on the comprehensive treatment in [2, 3, 4]. Crowd stampedes and panics often lead to fatalities. While sometimes the

Sex	$v_i^0[m/s]$	Purpose	$v_i^0[m/s]$
All	1.34	Leisure	1.10
Men	1.41	Shopping	1.16
Women	1.27	Commuting	1.49
		Business	1.61

**Table 1:** Data from Weidmann [1] on the average speeds of pedestrians, broken down by sex and by purpose of travel.

cause, such as a fire, is known at other times these appear without apparent disturbance. Stampedes lead to a breakdown of normal pedestrian behaviour starting with pedestrians trying to move at much higher speeds, pushing and coming into physical contact with other pedestrians. This leads to jams, clogging and uncoordinated crowd movement. Additionally people tend towards mass behaviour and alternative exits are often overlooked. During panics, pedestrians often try to exit through the entrance they used to enter the building even if other exits are closer [5, 6]. This makes the effect of communication on evacuation dynamics of great interest.

## 1.2 Modelling Approaches

Motion of pedestrians has been studied since the 1960's. There are numerous methods of modelling pedestrians including cellular automata, AI Models and social forces models.

### 1.2.1 Cellular Automata

In the Cellular Automata (CA) models, both time and space are discrete. The maximal observed pedestrian density is around  $6 P/m^2$  (where P stands for People). Therefore, a pedestrian occupies around  $40 \times 40 cm^2$  of space giving the cell size of this model. Each cell can then be either full (with a person) or empty and the interaction with the nearest neighbours determine the update rules. The particle in this model wishes to move in a desired direction. If the cell in this direction is free, then the particle moves forward. If it is occupied then it tries to move perpendicular to the direction of motion or diagonal. If several particles try to move to a cell then one is picked randomly.

### 1.2.2 Totally Asymmetric Exclusion Process

The general set up of the totally asymmetric exclusion process (TASEP) is identical to that of a CA model. It was first introduced to model protein synthesis in 1968 [7]. Over the intervening years, TASEP have been used to model variety of situations from biological transport process [8, 9] over vehicular traffic [10] to pedestrian motion.

The distinguishing feature of the TASEP is the updated rule: an agent is picked at random and moves to the cell in front of it if this is free. For a population  $N$ , one time step has passed once  $N$  updates have been performed.

### 1.2.3 Floor Field Cellular Automaton

The Floor Field CA model is identical to general CA models with the addition of a floor field. It was inspired by the pheromone trails used by ants to communicate whether they found food on the path they took [2]. The purpose of the floor field is to transform long-range interactions into local effects to enable easier computation. This is more efficient

in complex geometries as one does not have to check explicitly whether interactions are blocked by walls.

Space and time are discreet just as in the CA model. However, there is a transition amplitude to every neighbouring cell depending on the floor field that evolves with time. This floor field combines the desired direction, interactions with walls, obstacles and other pedestrians.

On short distances, pedestrian to pedestrian interaction is usually repulsive and implemented by the rule that each cell can only be occupied by a single particle. Over long distances, pedestrian to pedestrian interactions are often attractive i.e. it is advantageous to follow a crowd going in the same direction. This is represented as a local interaction by the floor field. The effects of the infrastructure are shown by a static floor field that is constant. A dynamic floor field records the traces left by pedestrians that influences the motion of subsequent pedestrians and can decay with time.

#### **1.2.4 Social-Forces Model**

The social forces model is a continuum model based on Newtons equation. As the name implies, the psychological motivation of individuals to move towards their target and avoid collisions are described by "social forces". The origin of these is of psychological rather than physical nature but they are otherwise treated identically. Additionally there are physical forces such as friction between objects and forces opposing compression. This model, proposed by Helbing and Molnár [11] will be the focus of our work and is described in detail in Section 2.1. It will henceforward be referred to as the Helbing Model with no intention of diminishing the contribution of the co-authors.

#### **1.2.5 Fluid Dynamic / Kinetic Models**

As pedestrian dynamics have similarities with fluids, some of the earliest approaches were based on kinetic theory, treating pedestrians as a gas. In these models the interactions between the pedestrians are modelled as collision processes with the exchange of momentum and energy. As we are not concerned with such models in this work, this will not be discussed in this report but Nishinari *et al* [2] and references therein extensively review these models.

### **1.3 Goal of the Project**

Our initial goal was to reproduced several recent findings from simulations done with CA models using the more complex Helbing Model. We aim to investigate the differences between simple CA models that aim only to capture the basic phenomenology to the Helbing model that attempts a more detailed description of the underlying processes. We considered the effect of communication/direction on evacuation in two scenarios: evacuation from a large room and exit choice at a T-Junction.

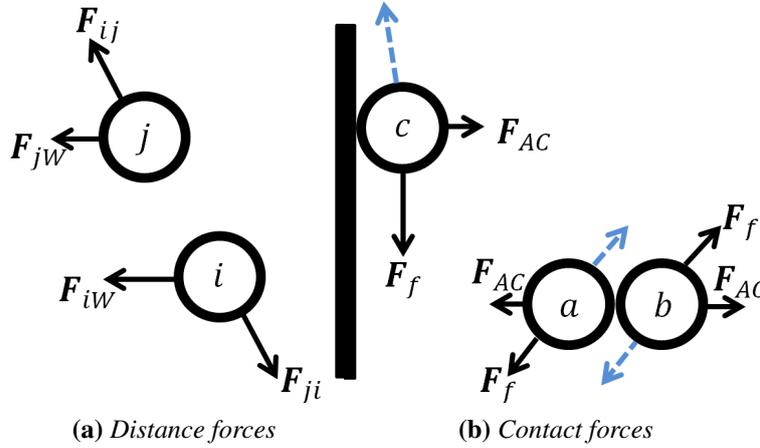
My main focus has been the effect of communication in pedestrian evacuation based on a paper by Smyrnakis and Galla [12]. In this context the agents use communication to effectively choose one of several exits from a large area such as a bridge. For this reason we refer to this as the bridge scenario. The results of our reproduction are detailed in section 3. My partner, Paul Vriend, focused on the behaviour of agents at a T-Junction. Galla [13] investigated this phenomena both analytically and using a CA model which

serve as a comparison to the Helbing Model. The results from this scenario, which we termed the T-Junction scenario, are given in section 4.

## 2 Reproducing the Helbing Model

### 2.1 Description of the Model

We have adapted the model Helbing, Farkas and Vicsek proposed in their Nature paper [5]. This model describes the natural tendencies of pedestrians to keep a distance from other people and walls and the wish to reach a target using forces. There are repulsive forces between agents and between agents and walls: pedestrians wish to avoid collision. If agents come into contact with each other or the walls, two additional forces arise. A frictional force, impeding tangential motion and a force opposing the compression of the agents. These forces are shown in Fig. 1.



**Figure 1:** Diagram of the forces that arise in the Helbing model. On the left hand side (a), distance forces are shown. The subscripts indicate the forces where  $W$  stands for the force exerted by the wall on the agents. On the right hand side (b) contact forces are illustrated (the distance forces that are present are not shown). The subscript  $AC$  indicates anti-compression forces and  $f$  indicate frictional forces. The dashed blue (colour online<sup>1</sup>) arrows indicate the velocity of the agents.

Each pedestrian  $i$  of mass  $m_i$  wishes to move in a certain direction  $\mathbf{e}_i^0$  with a desired velocity  $v_i^0$  and accordingly accelerates to adjust the actual velocity  $\mathbf{v}_i$  in a characteristic time  $\tau_i$ . At the same time the pedestrian tries to keep a distance from other pedestrians  $j$  and walls  $W$ . This is represented by the interaction forces  $\mathbf{f}_{ij}$  and  $\mathbf{f}_{iW}$  ( $\mathbf{F}_{ij}$  and  $\mathbf{F}_{iW}$  in Fig. 1). The equation of motion is given by

$$m_i \frac{d\mathbf{v}_i}{dt} = m_i \frac{v_i^0(t) \mathbf{e}_i^0(t) - \mathbf{v}_i(t)}{\tau_i} + \sum_{j(\neq i)} \mathbf{f}_{ij} + \sum_W \mathbf{f}_{iW} \quad (1)$$

where the change in position  $\mathbf{r}_i(t)$  is given by the velocity  $\mathbf{v}_i(t) = d\mathbf{r}_i/dt$ .

The avoidance of contact between pedestrians  $i$  and  $j$  is modelled by the force  $A_i \exp[(r_{ij} - d_{ij})/B_i] \mathbf{n}_{ij}$  where  $A_i$  and  $B_i$  are constants.

The distance between the pedestrian's centre of mass is  $d_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$  and the normalized vector pointing from pedestrian  $j$  to  $i$  is given by  $\mathbf{n}_{ij} = (\mathbf{r}_i - \mathbf{r}_j)/d_{ij}$ . If

<sup>1</sup>mphys.herobo.com

the distance between them,  $d_{ij}$ , is smaller than the sum  $r_{ij} = r_i + r_j$  of their individual radii  $r_i$  and  $r_j$ , the particles are in contact. In this case two additional forces come into play: a "body force"  $k(r_{ij} - d_{ij})\mathbf{n}_{ij}$  counteracting compression and a frictional force between the pedestrians  $\kappa(r_{ij} - d_{ij})\Delta v_{ij}^t \mathbf{t}_{ij}$  impeding relative tangential motion. The tangential direction is given by  $\mathbf{t}_{ij} = (-n_{ij}^2, n_{ij}^2)$  and the tangential velocity difference is  $\Delta v_{ij}^t = (v_j - v_i) \cdot \mathbf{t}_{ij}$ . In summary, we have

$$\mathbf{f}_{ij} = \left[ A_i \exp\left(\frac{(r_{ij} - d_{ij})}{B_i}\right) + k g(r_{ij} - d_{ij}) \right] \mathbf{n}_{ij} + \kappa g(r_{ij} - d_{ij}) \Delta v_{ij}^t \mathbf{t}_{ij} \quad (2)$$

where the function  $g(x)$  is zero if the pedestrians do not touch each other and is otherwise equal to its argument  $x$ .

The interaction with the walls is treated analogously. The distance to the wall  $W$  is  $d_{iW}$ ,  $\mathbf{n}_{iW}$  denotes the direction perpendicular to the wall and  $\mathbf{t}_{iW}$  the direction tangential to the wall. The wall force is then given by

$$\mathbf{f}_{iW} = \left[ A_i \exp\left(\frac{r_i - d_{iW}}{B_i}\right) + k g(r_i - d_{iW}) \right] \mathbf{n}_{iW} + \kappa g(r_i - d_{iW})(\mathbf{v}_i \cdot \mathbf{t}_{iW}) \mathbf{t}_{iW} \quad (3)$$

The constants were defined identical to those that Helbing *et al* used: a mass of 80 kg and an acceleration time of  $\tau_i = 0.5s$ . The constants  $A_i = 2 \times 10^3 N$  and  $B_i = 0.08m$  are defined such that the models reproduce the distances kept by crowds under normal desired velocities and fit the measured flows through bottlenecks. The parameters  $k = 1.2 \times 10^5 kg s^{-2}$  and  $\kappa = 2.4 \times 10^5 kg m^{-1} s^{-1}$  determine the friction and anti-compressional forces in case of contact. To avoid gridlock, some irregularity is introduced into the model by uniformly distributing the radii  $r_i$  of the pedestrians in the range [0.25m, 0.35m].

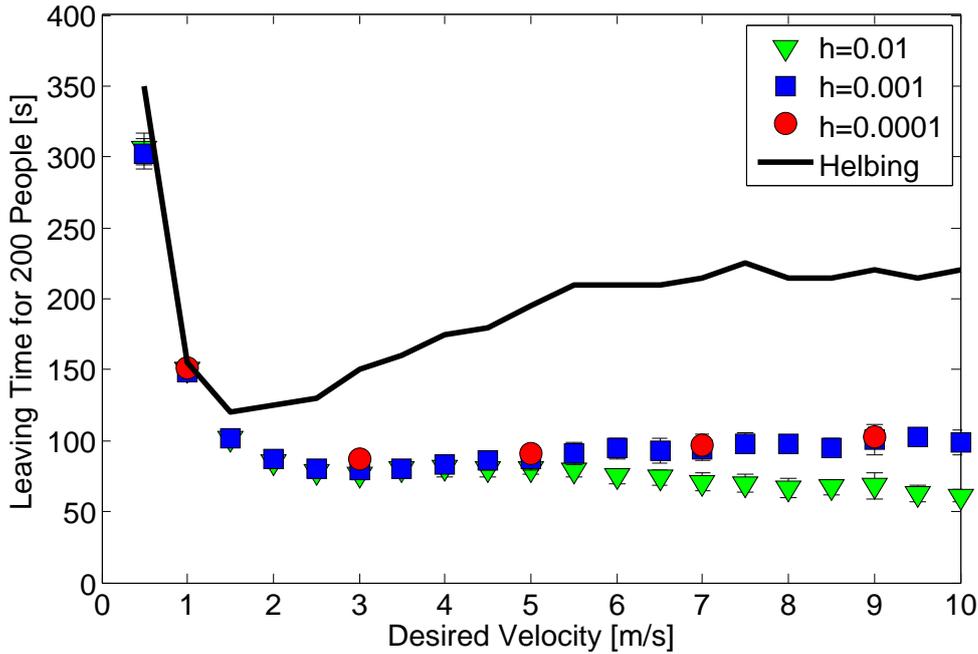
The equations of motions are integrated numerically using the Euler method. Taking an ordinary differential equation of the form  $dy/dt = f(t, y(t))$  subject to the initial condition  $y(t_0) = y_0$ . Let the step size be  $h$  which allows us to define time in terms of the initial time  $t_0$  by  $t_n = t_0 + nh$ . One step of the Euler method from  $t_n$  to  $t_{n+1} = t_n + h$  is given by

$$y_{n+1} = y_n + hf(t_n, y_n) \quad (4)$$

where  $y_n$  is the value of the function at time  $t_n$ , i.e.  $y_n \approx y(t_n)$ . The initial condition in our simulations is the initial position of the pedestrians in the room. These are set by randomly distributing the population of pedestrians over the available area while ensuring that no overlap occurs. The time step  $h$  determines the accuracy of the numerical solution. This is a trade off between computational speed and accuracy and we experimentally determined a step size where a further reduction did not alter the behaviour of the simulation.

## 2.2 Verifying our Model

Helbing *et al* calibrated their model against the experimental observation that 0.73 persons per second pass through a 1m wide door when the desired velocity is  $v_i^0 \approx 0.8ms^{-1}$  [1]. When simulating this scenario we found a value of  $0.8 \pm 0.1 P/s$ . To ensure that we reproduced the Helbing model we decided to reproduce the "faster-is-slower" effect Helbing *et al* observed when simulating escape from a room: when people wish to move faster through the exit, the increased pressure creates clogging that slows down the rate of escape.



**Figure 2:** Leaving time for 200 people from a square room of  $15 \times 15$  meters and a 1 meter wide door. The equations of motion are numerically integrated using the Euler method with a time-step  $h$  and averaged over 20 runs. The results for simulations with time steps of  $h = 10^{-1}$  (green triangles),  $h = 10^{-2}$  (blue square) and  $h = 10^{-3}$  (red circles) are shown. The results of Helbing et al are shown (black lines) [5].

The escape time versus desired velocity from a room of  $15 \times 15$  meters and a 1 meter wide door is shown in Fig. 2<sup>2</sup>. While we reproduce the "faster-is-slower" effect, it is much less pronounced and our minimum leaving time is lower than that of Helbing *et al*. There is a clear discrepancy between Helbing *et al* results and our result. There are several possible reasons for this discrepancy. For the model to work a velocity cut off is needed. Helbing *et al* describe this in their 2001 paper [14] but made no mention of it in their 2000 paper [5]. The use of a different velocity cut-off could explain the differences in our results. Another possibility is that the graph we used for comparison is a model that includes injury of pedestrians when the force on them exceeds a certain pressure. These injured people change the dynamics of the crowd making a direct comparison invalid. The caption in their 2000 paper is not explicit. While we could not reproduce the exact graph we have reproduced the "faster-is-slower" effect and fit our data to the observed rate through an exit 1m wide. Therefore we concluded that our model was working and proceeded to apply this to new scenarios.

### 2.3 Computational Optimization

Compared to a cellular automaton model, where the number of computations is of the order of the sample size  $\mathcal{O}(N)$ , in the Helbing Model the computational time is of the order of  $\mathcal{O}(N^2)$  as each pedestrian evaluates the force from every other pedestrian. The first step in the project was to choose a programming language to write in, our two main requirements being computational speed and the ease with which graphical output could be generated. We choose Java as it aids the implementation of graphical output and has

<sup>2</sup>Example videos of this and later simulations can be found on <http://mphys.herobo.com>

a comparable speed to native languages such as C++. Java has the added benefit that the code is transportable between Operating Systems without needing any adjustments.

To reduce calculation time, several optimizations were introduced such as a distance cut-off. If the distance to a person or a wall is greater than  $1.7m$  and hence the force from this object is less than  $10^{-3}N$ , the force is not calculated as the effect is negligible compared to other forces acting on a pedestrian.

We received access to the EPS high throughput computational facility running Condor. This enabled us to perform much wider parameter sweeps than would have been feasible with desktop computers. A total of 7000 hours of CPU time was clocked to obtain our results.

### 3 The Bridge Scenario

This scenario is based on a paper by Smyrnakis and Galla [12] in which they investigate the effects of communication and utility-based decision making on evacuation from a room. Several approaches to decision making exist, such as neural networks [15]. These decision making models typically consider information about the area that is being evacuated. Smyrnakis and Galla considered the effect of direct communication between agents on evacuation efficiency.

We replicated the set up and ran simulations with the Helbing Model to investigate whether their results persist with the change of model. The code for this scenario as well as running the simulations was my responsibility while my partner, Paul Vriend, concentrated on the T-Junction. However, throughout we consulted each other on problems and the best approach to the implementation of the code.

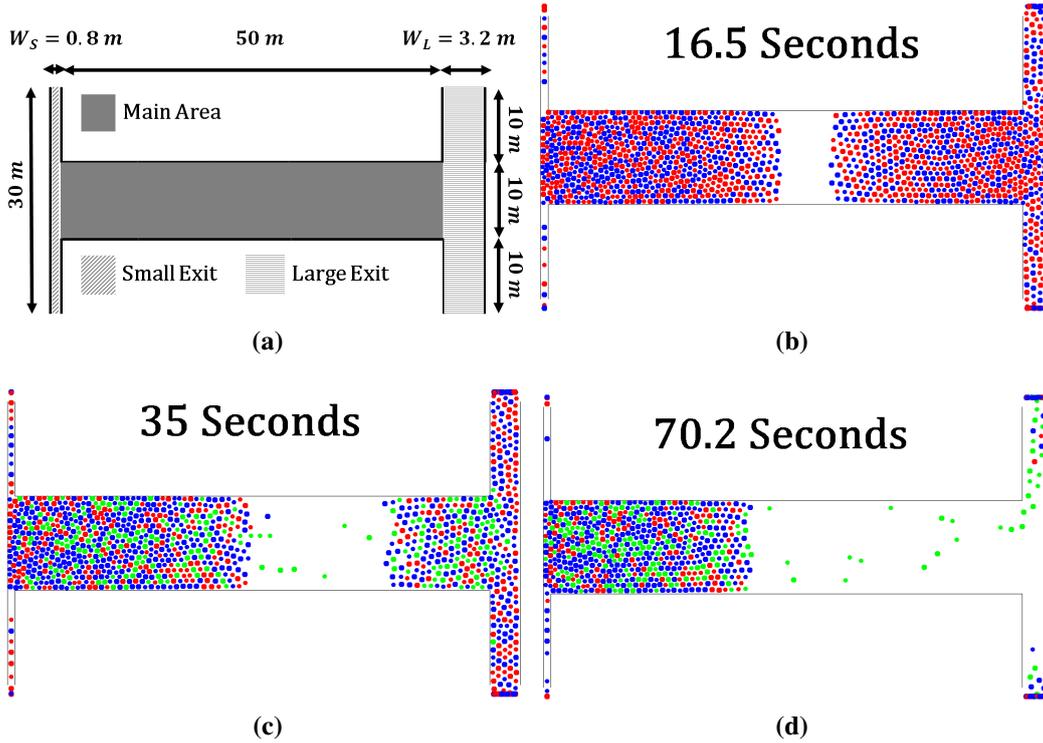
#### 3.1 Set Up

Smyrnakis and Galla use a CA model to propagate the particles. Their room has a large central rectangular area  $10m \times 50m$  with exits on the shorter sides. On each side there are two exits. The exits on one side have a width of 1.6 or 3.2 meters ( $L$  for "larger") while the exits on the other side are of the width of 0.8 meters ( $S$  for "smaller"). Once the particles leave the central area they move up or down, depending which exit is closer. A schematic and pictures from an evacuation are shown in Fig. 3.

Agents in the main evacuation area use a utility function to assign a utility to each exit and then move towards the exit with the highest utility. The default utility function is based on the expected time to reach a certain exit, i.e. agents move to the closest exit. Certain agents are given the possibility to communicate, meaning they can exchange information about which exit they are heading towards and their velocity. This could correspond to communication via a mobile phone. To this end they are randomly paired up at the beginning of the simulation and after a set interval  $t_I$  they start to exchange information. Assume that agent  $\alpha$  moves towards the exit  $S$  with velocity  $v_{\alpha S}$  and its partner  $\beta$  moves towards exit  $L$  with velocity  $v_{\beta L}$ . In this case the utility assigned by agent  $\alpha$  to the larger  $u_{\alpha L}$  and the smaller  $u_{\alpha S}$  exit are

$$u_{\alpha S} = \frac{r_{\alpha S}}{v_{\alpha S}}, \quad u_{\alpha L} = \frac{r_{\alpha L}}{v_{\beta L}} \quad (5)$$

where  $r_{\alpha S}$  and  $r_{\alpha L}$  are the distance from agent  $\alpha$  to the smaller and larger exit respectively. The velocity represents the instantaneous motion towards the target and we use the



**Figure 3:** *Schematic of the Bridge Scenario and 3 Screen shots from the simulation at 16.5, 35 and 70.2 seconds with a population of 1500 people with 50% of the population being able to communicate. Pedestrians coloured blue do not have a partner and pedestrians with a partner are coloured red. These turn green when they change their exit. (a) Schematic of the layout. (b) Simulation at 16.5 seconds, all pedestrians are moving towards their closest exit. (c) Simulation at 35 seconds, agents that are able to communicate. The majority of the agents that decided to change exits are trapped in the crowd (d) The larger exit has emptied and agents that are communicating with a partner that left through the larger exit are trying to move towards it but continue to be blocked*

average motion over the last second of simulation. Communicating agents can revise their decision after the interval  $t_I$  has passed. Henceforth we will refer to this utility function as the "Direct" utility function as agents assume they can travel in a straight line to all exits.

Smyrnakis and Galla observed that by increasing the fraction of people communicating the evacuation time decreased until 60% of agents communicate. Any further increase in communication lead to a increase in evacuation time. Their original graph is show in Fig. 4(a)

An initial question in reproducing Smyrnakis and Galla communication was what part of their model was due to the discreet nature of the CA and what was a description of the underlying physics. We implemented the exact utility function used by Smyrnakis and Galla with the change that agents moving directly to the nearest exit as diagonal motion is possible in the Helbing model (simulations following the exact implementation of Smyrnakis and Galla showed no difference in behaviour to this implementation). Additionally, we introduced a changed utility function. Instead of assuming that from the current position an agent could move to the alternative exit at its partners velocity, we introduced a utility function where the agent assumes he can travel at his desired velocity to the other agents position, and from the other agents position at its speed to the exit. Therefore, the

utility  $u_{\alpha S}$  and  $u_{\alpha L}$  assigned by agent  $\alpha$  to the smaller and larger exit respectively is given by

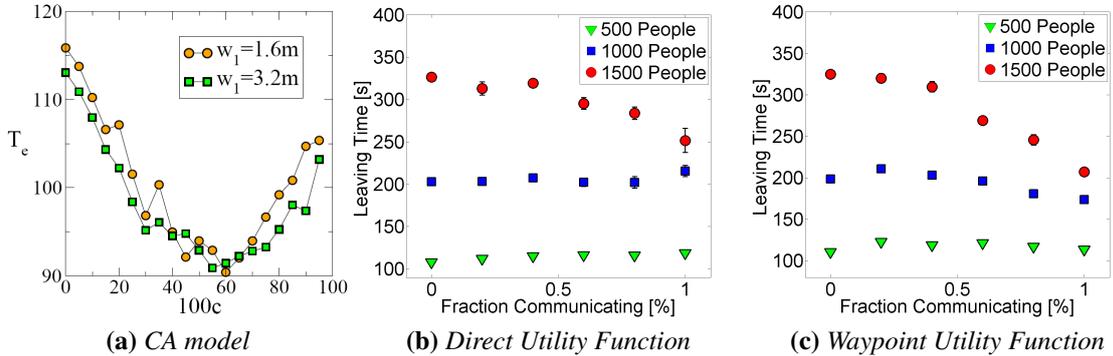
$$u_{\alpha S} = \frac{r_{\alpha S}}{v_{\alpha S}}, \quad u_{\alpha L} = \frac{r_{\beta L}}{v_{\beta L}} + \frac{r_{\alpha\beta}}{v_{\alpha}^0} \quad (6)$$

where the agent  $\beta$  is the partner of agent  $\alpha$  as before. Henceforth we will refer to this as the "Waypoint" utility function as the agents estimate the time with their partners position as a waypoint.

Smyrnakis and Galla used a population of 2100 people for their simulations. For the simulations with the Helbing model, the largest population we used was 1500 people. This corresponds to approximately 85% of the surface of the room being covered by agents. While the number of agents is smaller in our simulations than in those of Smyrnakis and Galla, in our model the agents cover a larger surface area than 2100 people in the CA model, where they cover 67%. Furthermore, the square nature of the cells in the CA model mean it is possible to cover the entire surface area with agents while in the Helbing model this is impossible. The maximal packing of circles in a plane is 90.7%. As in our simulations agents are placed randomly, this is unlikely to be achieved and therefore 85% is close to the practical maximum.

## 3.2 Results of the 1st Semester

### 3.2.1 Agent-to-Agent Communication



**Figure 4:** The effect of phone communication in the bridge scenario. The graphs show the leaving time versus the fraction of the population communicating. The data is averaged over at least 20 replications with the larger exit width at 3.2m. (a) The results of Smyrnakis and Galla using a CA model [12] with 2100 agents.  $w_1$  refers to the width of the larger exit. (b) Simulations with the Helbing model using the Direct Utility function (identical to Smyrnakis' and Galla's) (c) Simulations with the Helbing model using the Waypoint Utility function.

The leaving times for the two utility functions are shown in Fig. 4 along with the original results from Smyrnakis and Galla (Fig. 4a). It is apparent that none of the simulations with the Helbing Model reproduce the originally observed minima in evacuation time.

The CA model predicts much shorter evacuation times with no communication than the Helbing model, 113 seconds for 2100 pedestrians compared to 370 seconds for 1500 pedestrians in the Helbing model. This is due to the ability of the Helbing model to simulate arching and clogging at exits. Two agents trying to enter an exit may block

each other moving forward until the pressure from the crowd behind them breaks the symmetry of forces such that one can advance. This can lead to prolonged blockages of the exit unlike in the CA model where one agent always is able to move to a free cell.

The larger the population is the more influence communication has. For a population of 500 people around half will have left the structure by the time agents start communicating and the time to travel to the free, larger exit is almost equivalent to the time until the smaller exits becomes free. For a population of 1000 people communication can slow as well as speed up evacuation times, depending on the initial conditions, i.e. the placement and the pairing of the communicating agents. On average it speed up the evacuation. For 1500 people communication always improves evacuation times.

All simulations with the Helbing model show the same general trend, a reduction of evacuation time with increasing communication. The lowest evacuation time occurs when all pedestrians are communicating and this effect is most pronounced for larger populations. However, this does not concur with the findings of Smyrnakis and Galla. While the simulations with the CA model show an improvement in evacuation time with increasing communication, the minima they observed is not reproduced with the Helbing model. The overall larger evacuation time with the Helbing models is most likely due to the more realistic occurrence of gridlock and congestion in this model.

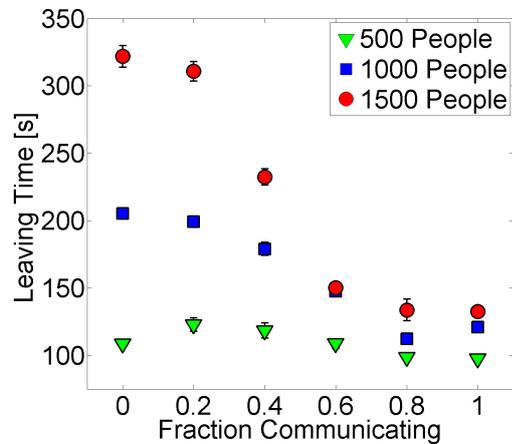
While the two utility functions cause differences in the behaviour of the system, the qualitative behaviour remains the same. This shows that the results are not dependent on the exact implementation of the utility functions and hence robust to this model parameter.

### 3.2.2 "Text Messaging" Communication

We had two motivations for introducing an alternative form of communication. The drawback of the abstraction of the Smyrnakis and Galla model is that there is no one-to-one translation to real evacuation situation. Groups and people that know each other generally stay together and pedestrians display herding behaviour [14]. It is therefore unlikely for a large fraction of pedestrians in a given evacuation to be able to communicate with a partner in a different exit. Additionally, we were interested in a model that allows us to influence the number of pedestrians going to each exit more directly. To this end we introduced what we term the "Text messaging" model. A fraction of the population, randomly selected, is presented with the instructions that to go to one of the two large exits. If the agent in question is already using this exit then this does not change its motion. Likewise, if the agent has left through one of the exits already, it does not revise its choice. The default choice of the agents is to move towards the closest out of the four exits.

To this end we introduced what we term the "Text messaging" model. A fraction of the population, randomly selected, is presented with the instructions that to go to one of the two large exits. If the agent in question is already using this exit then this does not change its motion. Likewise, if the agent has left through one of the exits already, it does not revise its choice. The default choice of the agents is to move towards the closest out of the four exits.

In reality pedestrians would have a certain propensity to follow an instruction from e.g. a text message sent by a central system. However, as this just scales the overall



**Figure 5:** The effect of communication by a central authority in the bridge scenario. The graphs show the leaving time versus the fraction of the population being directed towards the larger exits. The data is averaged over at least 20 replication.

number of people following external advice we did not include a probabilistic propensity in our model. This would introduce a further stochastic variable and the model already has a stochastic element in it as the initial position of pedestrians is randomly assigned. This introduces the constrain on our results at large percentages of "texted" people will not be reproducible in real life. The actual propensity to follow is an experimental constant to be determined. In the meantime, this model allows the investigation of the effect of a central directing agent on evacuation flow.

The results are shown in Fig. 5. For large enough populations, directing the agents always leads to an improvement in evacuation time. The slight rise in evacuation time observed for a population of 500 people is caused by the slow-down due to counter flowing pedestrians.

It is notable that for the fraction communicating  $\geq 0.6$  the evacuation time is around 150 seconds independent of the population size. This stems from the inefficient default choice of a quarter of the population going through each exit even though the larger exits have the fourfold capacity of the smaller ones. However, this is in accordance with observed evacuation behaviour where pedestrians tend to use the door they used to enter the structure to exit, even if closer exits are available.

This method of communication shows the largest decrease in evacuation time, 50% for a population of 1500 people.

### 3.3 Results

In the first semester the focus of our work was to implement the Helbing model and validating it. We investigated the behaviour of this model but were hampered by a lack of computational resources. This has changed in the second semester and has allowed us to investigate the behaviour of the Helbing model over a wider parameter range.

The result of the 1st semester show that the effects of communication are most pronounced with large populations. Therefore, in the 2nd semester we have only used a population of 1500 people to limit the number of simulations needed.

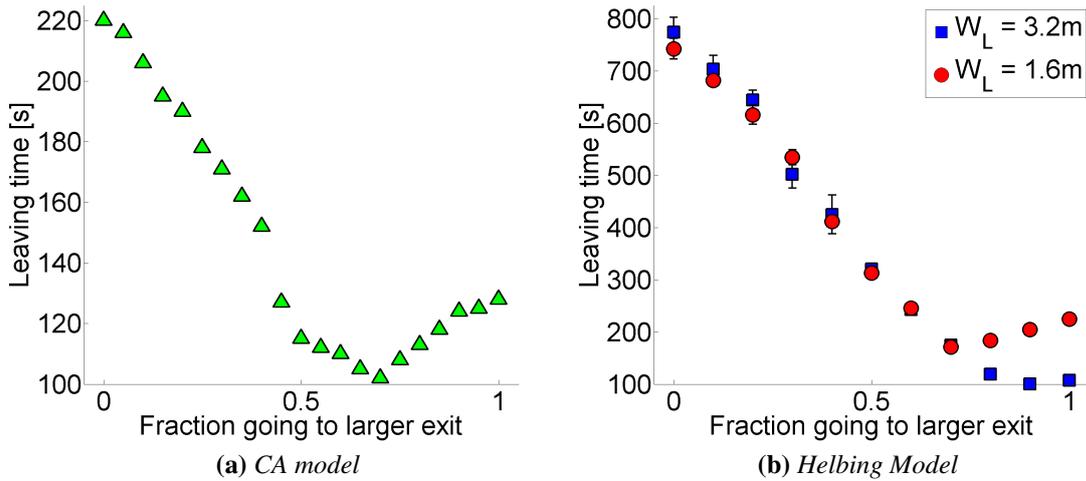
#### 3.3.1 Behaviour without communication

At the start of this semester, we set out to find the optimal fraction of agents to use the larger exits. This provides information on how much change communication needs to effect to bring the system from its starting state to an optimal state. For a population of  $N$  agents, the fraction  $pN$  going to the larger exit was changed and the result on leaving time measured, as shown in Fig. 6. As Smyrnakis and Galla [12] have include this data in their paper this is a further point of comparison for the two models.

For identical exit widths the optimal strategy for the CA is  $p = 0.7$  while it is  $p = 0.8$  for the Helbing model. It is apparent that clogging and mutual impeding of agents plays a far larger role in the Helbing model where evacuation through the smaller exits take 3.7 times as long than through the larger exit compared to 1.7 for the CA model. As the width of the larger exits is increased, it becomes optimal to send a larger population to the larger exit.

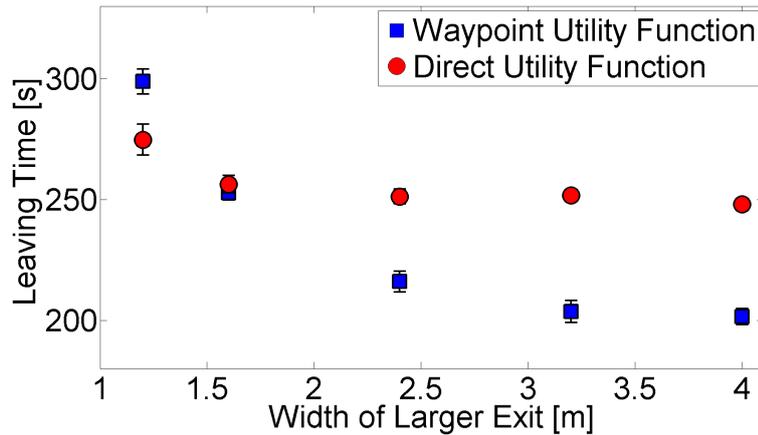
#### 3.3.2 Varying the Width of the Larger Exit

For the investigation of the effect of exit width, the remaining model parameters were fixed. The communication interval was set to  $30s$ , the smaller exit to  $0.8m$ , the fraction



**Figure 6:** Effect of changing the fraction of the population going to the larger exit on leaving time. **a)** Results for CA model and an exit width of 1.6m, adapted from [12] **b)** results for the Helbing model for two exit widths, 3.2 (squares) and 1.6 (circles) meters.

of communicating agents to 1 and the population to 1500 agents. The results are shown in Fig. 7.



**Figure 7:** The effect of varying the exit width on leaving time for the two utility functions. The communication interval is set at 30s, all agents are communicating and the smaller exit is fixed at a width of 0.8m

As the width of the larger exit increases, the evacuation time falls until a minima is reached. Further increases in the exit width beyond this maximal effective width have no effect. This occurs at 1.5 and 3 meters for the Direct and the Waypoint Utility function respectively.

It is apparent that the Waypoint Utility function has an optimum evacuation time 50 seconds faster than that of the Direct Utility function for a large difference between the exits. For a larger difference between the small and the large exit a change to the larger exit is more likely to improve evacuation time. By the time an agent has reached the larger exit, most pedestrians will already have escaped and hence the situation is generally better than expected from the communication the agent received. The Waypoint utility function assigns a larger utility to changing and hence is more effective under this circumstance. For this reason it is less effective than the Direct utility function at small differences in

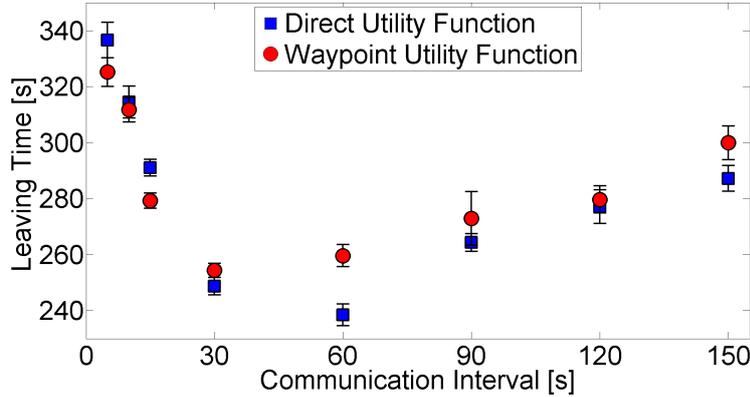
exit width between the small and large exit.

We have determined that the maximal effective width for the Direct Utility Function is 1.5 m. If this applies to the CA model it explains why Smyrnakis and Galla find no difference between an exit width of 1.6m and 3.2m, as both of these lie above the maximal effective width.

For our subsequent investigations the larger exit width was fixed at 1.6m. It was fixed to limit the number of parameters under investigation and 1.6 meters were chosen to allow easy comparison to Smyrnakis and Galla’s results.

### 3.3.3 The Effect of the Communication Interval

A parameter sweep over the communication interval was conducted. The number of agents leaving through each exit along with the number of times communicating agents change exits was recorded for different communication intervals. The effect of the communication interval is show for all agents communicating in Fig. 8 and the number of agents changing exits is shown in Fig. 9. A set of graphs with intermediate fractions of agents communicating is shown in Appendix A.



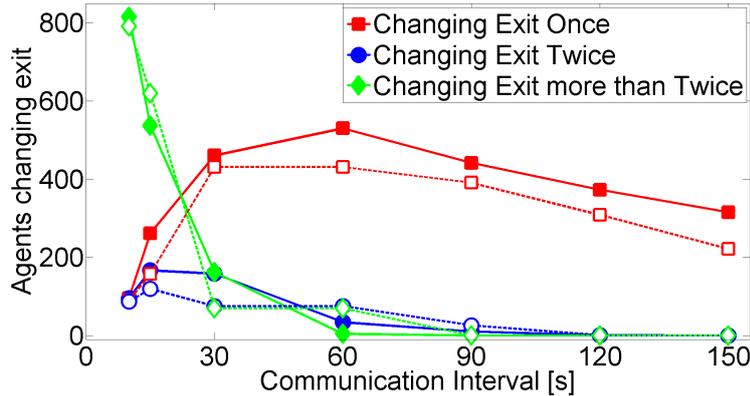
**Figure 8:** Varying the communication interval with a population of 1500 agents, exits widths of 0.8 and 1.6m respectively with all agents communicating. Each data point is averaged over 20 trials.

At short communication intervals ( $< 30$ s), communication has no or a detrimental effect. This is due to the counterflow of agents changing exits. Especially those changing exits multiple times create jams. The number of agents changing exit more than twice drops steeply from the majority at communication intervals of 10 seconds to none above 30 seconds.

At intermediate communication intervals (30s - 60s), a minima in evacuation time occurs together with a maxima in agents changing exits once. For communication intervals above 30 seconds only small number of agents change more than once. As the communication interval increases above 60 seconds, the evacuation efficiency starts to drop as the communication occurs too late to have an optimal effect on the evacuation of the system.

## 3.4 Comparing the CA and Helbing Model

Smyrnakis and Galla present data for communication intervals of 30 seconds, which we investigated last semester and is shown in Fig. 4a. While the general reduction in evacuation time was reproduced in the Helbing model, not detrimental effect was observed at



**Figure 9:** The number of agents who change exits once, twice and more than twice out of a population of 1500. Filled markers are for the Waypoint Utility function and empty markers for the Direct Utility function.

large fractions of communicating agents. They have also included data for a communication interval of 13 seconds which is shown in Fig. 10 alongside simulations we now have done with the Helbing model.

It is apparent that while the Helbing model shows the same general trend of decreasing evacuation time as with longer communication intervals, the CA model show no or a detrimental effect of communication at this communication interval.

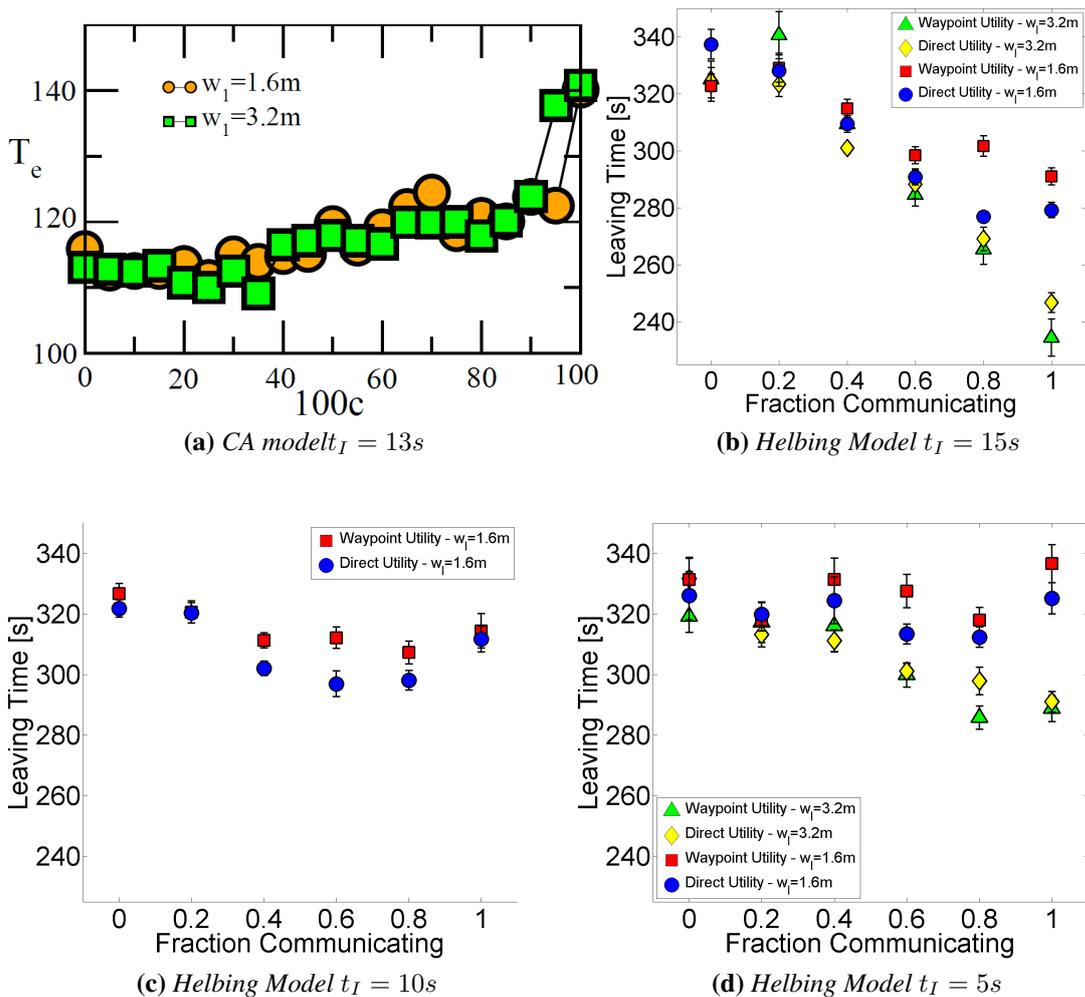
A possible reason is that in simulations with the CA model, time is measured in time steps. Smyrnakis and Galla have translated time steps to seconds by assuming that an agent moves at  $1.2 m/s$  and hence one second has 3 time step. However, this might not be directly comparable to time in the Helbing model and require a scaling factor. For this reason the results for the Helbing model are also shown at 10 and 5 seconds.

It can be seen that the results with the Helbing model at a communication interval of 5 seconds approximately match those of the CA model at 13 seconds. Similarly, the results for the CA model at a communication interval of 30 seconds (Fig. 4a) display a minima at 60 % of agents communicating similar to that of the Helbing model at a communication interval of 10 seconds. However, the underlying dynamic of the two model is distinct. The number of changes agents perform in the CA model for a communication interval of 13 and 30 seconds are shown in Fig. 11 a) along with our result for the same parameters. Note that the meaning of the filled and unfilled markers is different in Fig.11a) to Fig.11b)-d).

For a communication interval of 30 seconds, the number of changes of exits of the CA model and the Helbing model qualitatively correspond. A large number of agents changes exits once (40% and 30% respectively with all agents communicating) while fewer change twice or more times. However, in simulations with the Helbing model, the number of agents that change twice and more than twice is of the same order while in simulations with the CA model 2 changes are more likely than three or more changes. This stems from the fact that increased jamming and evacuation time in the Helbing model cause more agents to remain in the structure to communicate a third time.

For a communication interval of 15 seconds, no such qualitative correspondence is observed. In the Helbing model, most communicating agents change three times while in the CA model two changes predominate at small fraction of communicators and one change at a large fraction of communicators.

Considering the possibility that CA time needs to be rescaled by a factor of 3 to match Helbing time cannot be confirmed by this data. The behaviour of the CA model



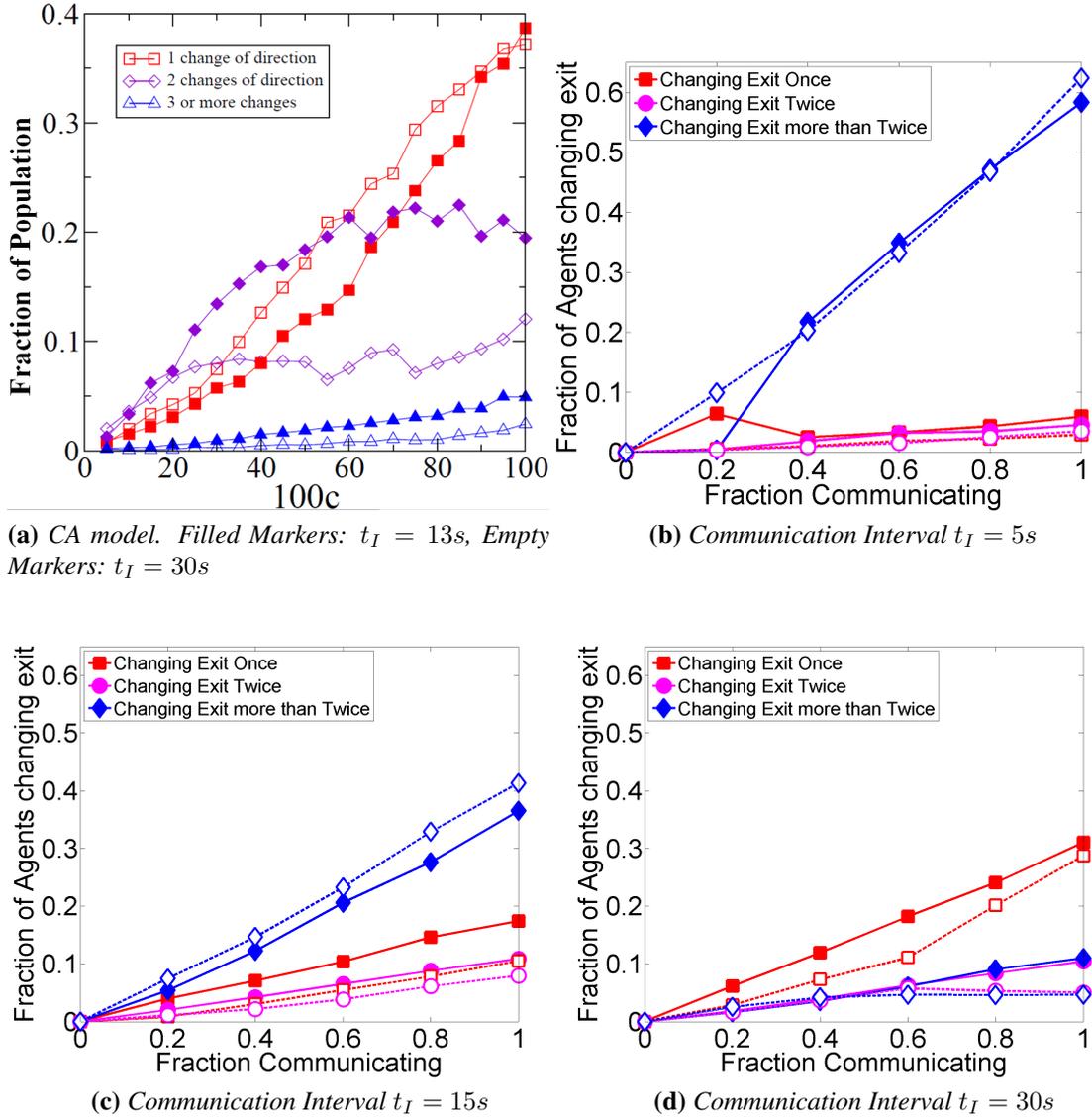
**Figure 10:** The leaving time as a function of the fraction of agents communicating. **a)** Simulations with the CA model and a communication interval of 13 seconds with a population of 2100 agents. Adapted from Smyrnakis and Galla [12]. **b)-d)** Simulations with the Helbing model for a communication interval of 15, 10 and 5s and a population of 1500 agents.

at a communication interval of 13 seconds is different to the Helbing model both at a communication interval of 15 and 5 seconds.

While we have observed a minima in leaving time for 60 % of agents communicating with the Helbing model (at a communication interval of 10 seconds), the dynamics do not match those of the CA model. In the CA model, the majority of communicating agents changed exits once while in the Helbing model, the dynamics at  $t_I = 5\text{s}$  is dominated by agents changing exits three or more times.

It also should be noted that at communication intervals of 15 and 5 seconds, there is a marked difference between an exit width of 1.6 and 3.2 meters even for the Direct Utility function. Earlier we found that for a communication interval of 30 seconds, no such difference exists.

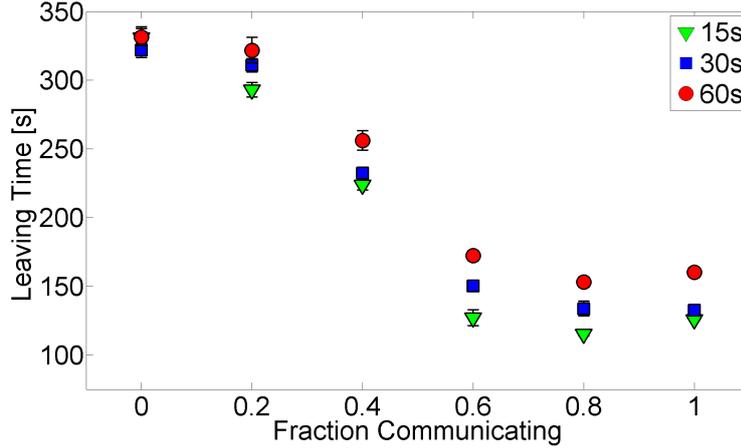
In summary it appears that there are fundamental differences in the simulation depending on which model is used. While the general observation that agent to agent communication improves evacuation efficiency is confirmed, the specific result that 60% of agents is optimal could not be confirmed. For a better understanding of the differences be-



**Figure 11:** The number of agents that change exits once, twice or more than twice. *a)* Results from Smyrnakis and Galla, adapted from [12]. Filled markers correspond to a communication interval of 13 seconds, empty markers to a communication interval of 30 seconds. *b), c), d)* Communication Intervals of 15, 30 and 60 seconds respectively with the Helbing model. Filled markers are for the Waypoint Utility function and empty markers for the Direct Utility function.

tween the models, investigating long communication intervals with the CA model would be of interest. However, due to time constraints this has to be left as a point of future investigation.

The large number of variables in the Helbing model make a complete understanding of the dynamics computationally costly. The population size remains a quantity that has not definite translation between the models. The effect of the desired velocity and the exact geometry remain unexplored. Considering these difference, the degree of similarity of the dynamics of the two models is surprising.



**Figure 12:** *The effect of communication by a central authority (“Text messaging”) in the bridge scenario at three communication intervals. The graphs show the leaving time versus the fraction of the population being directed towards the larger exits.*

### 3.5 “Text Messaging” Communication

Analogous to the investigation of the agent-to-agent communication, we were interested about the effect of the communication interval on the dynamics of the text messaging model. To this end we investigated the behaviour at three intervals, 15, 30 and 60 seconds with a population of 1500 agents. The larger exit width was fixed at 3.2 meters. The results are shown in Fig. 12.

A shorter communication interval (15s) leads to the best outcome although the differences between the communication intervals is below 15 %. Based on observing the dynamics, the shorter communication interval leads to the best outcome because the earlier redirection of agents to the large exits both lessens the jamming at the smaller exits and leads to a more efficient use of the larger exit.

## 4 The T-Junction Scenario

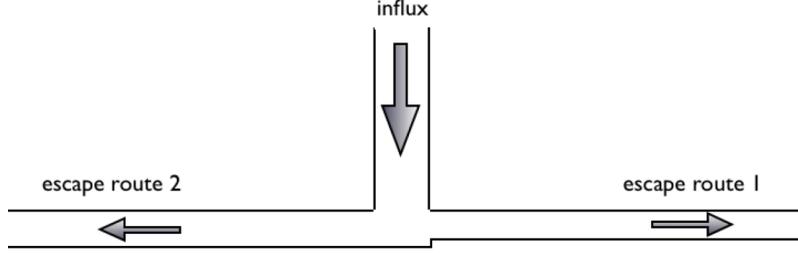
This scenario is based on a paper of Galla [13] and considers the case of a stylized intersection. People encounter a junction with two exits with unequal capacity. This scenario is solved analytically and the solution are compared to simulations with a cellular automaton model. We have reproduced simulations of this scenario with the Helbing model to see whether the behaviour persists under a change of model.

The model of pedestrian propagation used by Galla is that of a totally asymmetric exclusion process (TASEP). Complete solutions for the phases of this model have been found [16] allowing the phases of two TASEP, coupled over the central injection site, to be determined [13].

The code for this scenario as well as running the simulations was the responsibility of my partner, Paul Vriend. However, throughout we consulted each other on problems and the best approach to the implementation of the code.

### 4.1 Set up

The layout of the T-Junction is illustrated in Fig. 13. Particles are injected at the central site: if the area is free a new particle is inserted with a probability  $\alpha$ . The particles leave

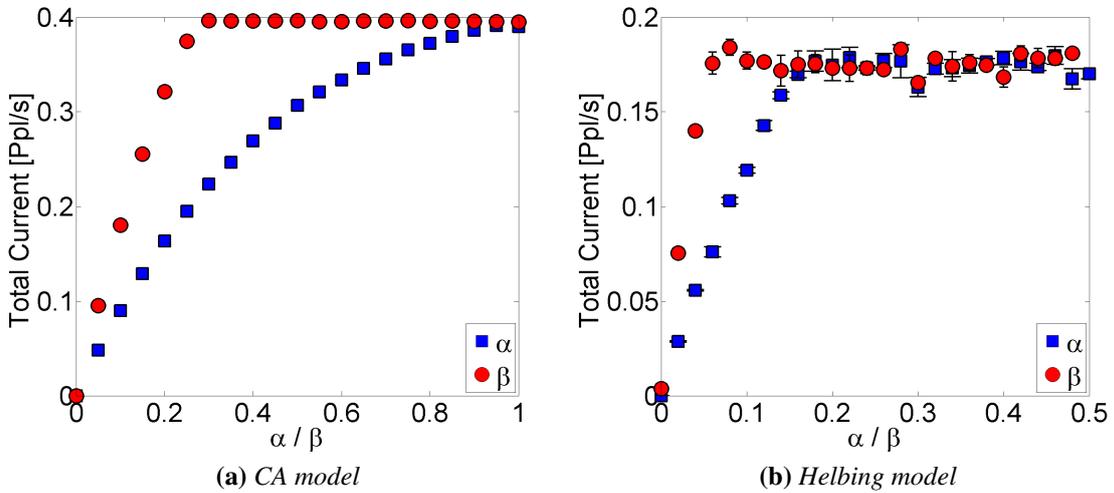


**Figure 13:** Layout of T-Junction scenario. The arrows indicate the direction of motion. Pedestrians are injected with a rate  $\alpha$  at the junction and move right with a probability  $p$ . The right hand side corridor is denoted the positive corridor and agents exit it with a probability  $\beta_+$  and the left hand corridor correspondingly with  $\beta_-$ . The image was adapted from Galla [13]

the system at the end of the corridors with a probability of  $\beta_+$  and  $\beta_-$  respectively. A final model parameter is the probability with which the pedestrians turn right,  $p \in [0, 1]$  and hence enter the left-handed branch with a probability of  $1 - p$ . In the simulations with the cellular automaton the parameters are all in the range  $[0, 1]$  per time step. This requires calibration for the Helbing model as it is continuous in space and time and hence it is not clear how to convert this to a rate per second.

The two branches of the system only differ by the respective values of  $\beta_{\pm}$ . These could correspond to the capacity of the escape route or the filling rate of a reservoir.

## 4.2 Calibration



**Figure 14:** Calibration of  $\alpha$  and  $\beta_{\pm}$  for both the CA models, where the quantities  $\alpha$  and  $\beta_{\pm}$  are clearly defined, and the Helbing model. Where the error-bars are not visible they are smaller than the data points.

In the original formulation of the model, all probabilities were defined in the range  $[0, 1]$  per time step of the model. As the Helbing model is effectively continuous in time we calibrated the  $\alpha, \beta_{\pm}$  model parameters. For some given dimensions of the junction, two were set to a large value ( $10^9$ ) and the parameter to be calibrated was increased from zero until it reaches the value  $V_{max}^H$  where a further increase does not lead to an increase in the current through the system.

We have used two means of calibration. Initially, we assumed that the value at which

the parameter plateaus in the Helbing model is equal to the maximum in the CA model, 1. This gives  $\alpha$  in the range  $[0, 0.15]$  and  $\beta_{\pm}$  in the range  $[0, 0.07]$ . We will refer to this calibration procedure as the "short" calibration.

For the second means of calibration, an identical procedure to that with the Helbing model was done with the CA model of the T-Junction to determine at what value  $V_{max}^{CA}$  is reached. It is then straightforward to map the values used by Galla in the cellular automaton to the corresponding values for the Helbing model. The calibration of the parameters  $\alpha$  and  $\beta_{\pm}$  are shown in Fig. 14. Therefore, by this method  $\alpha$  in the Helbing model is in the range  $[0, 0.15]$  and  $\beta_{\pm}$  in the range  $[0, 0.5]$ . We will refer to this calibration procedure as the "long" calibration.

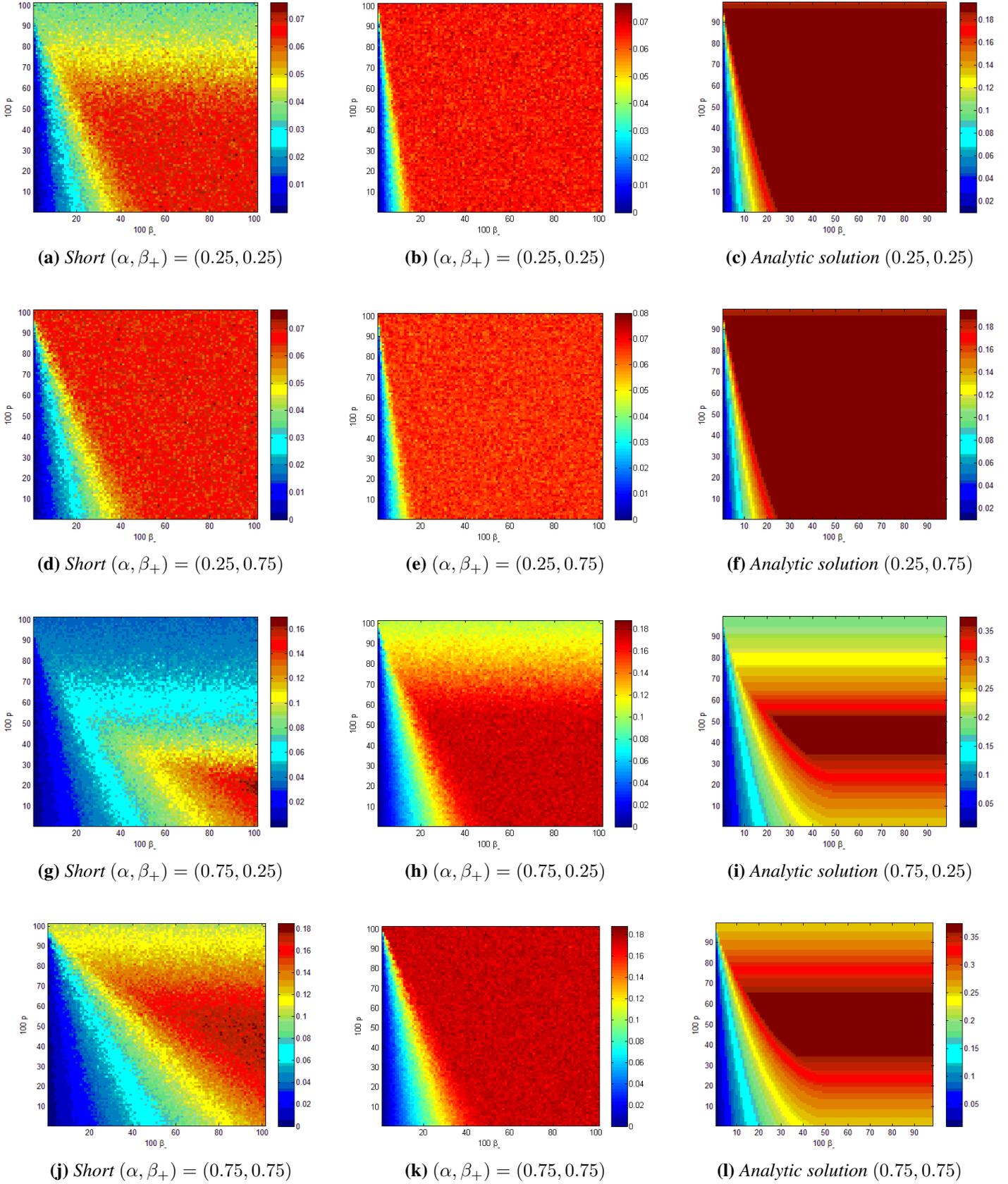
### 4.3 Results

We measured the total current through the system while changing the 4 parameters. To reduce computational time we restricted the parameter space by fixing  $\alpha$  and  $\beta_+$  and varying  $\beta_-$  and  $p$ . The results for four combinations of  $\alpha$  and  $\beta_+$  are shown in Fig. 15 for both methods of calibration along with the analytic solutions on the right hand side. Each heatmap is the average over 3 simulations where each simulation was run for  $10^4$  time steps of which the first 3000 were discarded.

For all four cases, the current in the analytic solution is increased by a factor of 2 compared to the Helbing model results. This ratio remains even though for  $\alpha = 0.25$  the current is approximately half the current in the  $\alpha = 0.75$  case, for both the analytic and Helbing results. This most likely stems from a difference in the corridor length. In the simulations with the CA model, the corridor were set to a length of 100 cells while for the Helbing model it was set to an equal length. As the agents in the Helbing model are larger, this corresponds to approximately 30 "agent-lengths".

For  $\alpha = 0.25$  one can see a good qualitative agreement between the simulation results with the long calibration and the analytic solution. For the short calibration, at  $(\alpha, \beta_+) = (0.25, 0.25)$  there is a reduced current at large values of  $p$ . As the short calibration has smaller values of  $\beta_{\pm}$ , this means a larger current is entering the system than can leave it, suggesting that the calibration of  $\beta_{\pm}$  is too "small", i.e.  $\beta_{\pm} = 0.25$  in the short calibration corresponds to a value lower than 0.25.

For  $\alpha = 0.75$ , the prominent features seen in the analytic results at low values of  $\beta_-$  are qualitatively present in the simulations results with the short calibration, Fig. 15g) & j). The fact that the analytic solution has less than the maximum current at large values of  $p$  indicates that the ejection rate of the positive branch is insufficient to match injection rate. This is not observed with the long calibration, indicating that  $\beta_{\pm}$  is calibrated too "large", i.e.  $\beta_{\pm} = 0.75$  in the long calibration corresponds to a value larger than 0.75.



**Figure 15:** Heatmaps showing the total current through the T-Junction as a function of  $p$  and  $\beta_-$  for four values of  $(\alpha, \beta_+)$ . On the left hand side the Helbing simulations with the short calibration are shown. In the middle, the Helbing simulations with the long calibration are displayed. On the right, the analytic results are shown, which have been adapted from Galla [13]

One can see a qualitative correspondence between the results of the Helbing simulations and the analytic solution. On one hand, given the drastic change in model from a TASEP to the Helbing model, this is surprising. On the other hand, the dynamics are severely restricted to an narrow corridor, meaning that it is impossible to fully see the difference between the models in this scenario. The ideal calibration of the Helbing model appears to lie between the short and long calibration.

## 5 Summary and Outlook

We have investigated the differences between reductionist cellular automaton (CA) models and the more detailed approach of social-forces (Helbing) model. In the case of evacuations from a large structure (the "Bridge" scenario) we have confirmed the observation that communication improves evacuation times for large pedestrian densities.

The result reported by Smyrnakis and Galla [12] that there is an optimal fraction of communicating agents was confirmed by us using the Helbing model. However, the underlying dynamics of the model causing this minima was different. Furthermore, after investigating a large number of communication intervals, we conclude that a larger fraction of communicating agents aids evacuation unless the communication interval is so small ( $15s > t_I$ ) as to be unlikely every to be attained in practice. We only reproduced their finding for a very limited set of communication intervals ( $5s < t_I < 15s$ ).

A point of interest would be to repeat the simulations at long communication intervals with the CA model to investigate in more detail the similarities and differences between it and the Helbing model.

Introducing a central steering agent, that directs pedestrians towards the larger exits, leads to better evacuation times than any agent to agent communication. This is also the measure most easily translated to a real live application.

For the case of agents encountering a junction ("the T-Junction scenario") we found agreement between the prediction of a totally asymmetric exclusion process (TASEP) and the simulations with the Helbing model. This is less surprising than for the Bridge Scenario as the restrictive geometry of the scenario means that and model of propagation will behave similarly.

A more realistic implementation of the T-Junction where the width of the exit corridors determines the capacity of each exit remains unfinished due to a lack of time. This would be a point for future work.

We have implemented the Helbing model and investigated two scenarios which have previously investigated with a CA model. The increased detail of the model (and accompanying large increase in computational cost) have yielded the generally the same behaviour. While practicalities prevent a real-live experiment, this result supports the notion that a lot can be learned about complex systems with minimalist, stylized models. However, this remains a trade off between realism and computational cost.

## Acknowledgement

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## Appendix A Communication Interval in the Bridge Scenario

The graphs showing the behaviour of the Helbing model at intermediate fractions of agents communicating and are shown here for completeness. The smaller and larger exit widths are 0.8 and 1.6 meters and the population is 1500 agents. In the left hand graph the leaving time, averaged over 20 runs is shown while on the right hand graph the number of agents changing exits once, twice and more than twice is displayed. For the changing exit data, the error was below 5% for all data points.

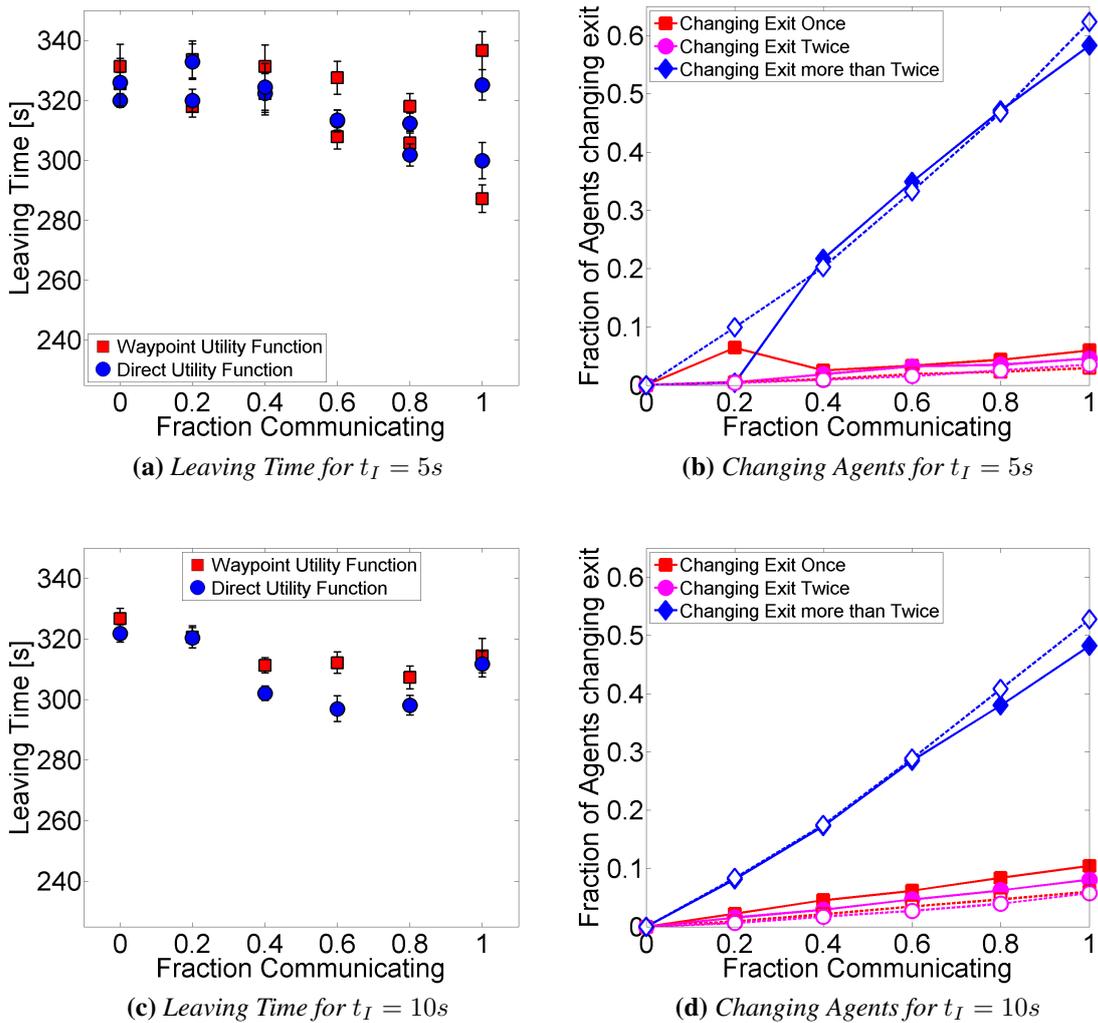
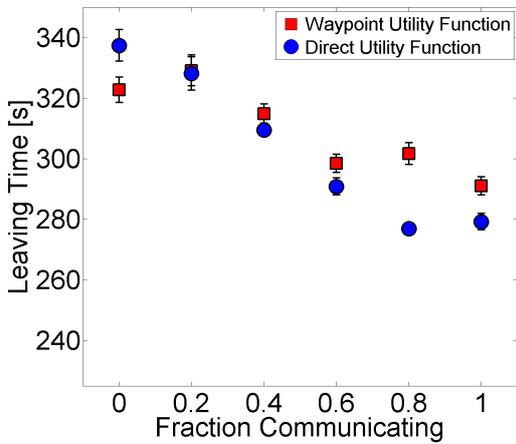
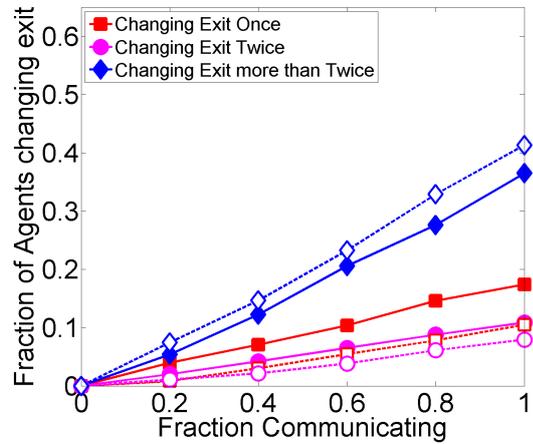


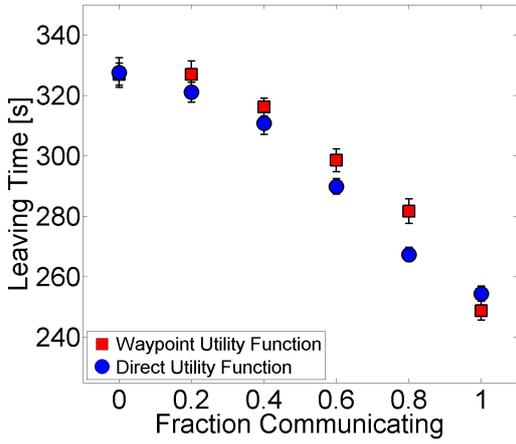
Figure 16



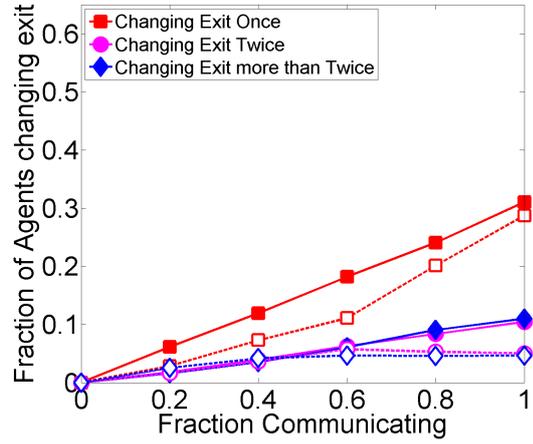
(a) Leaving Time for  $t_I = 15s$



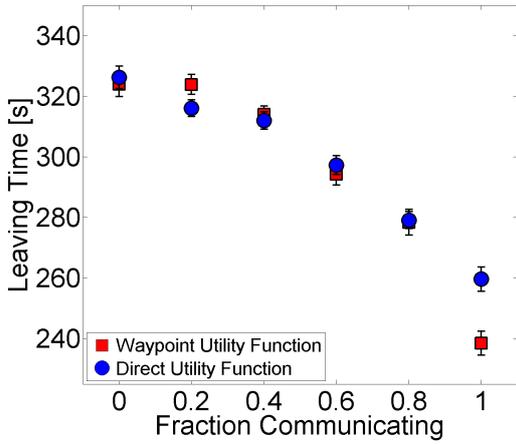
(b) Changing Agents for  $t_I = 15s$



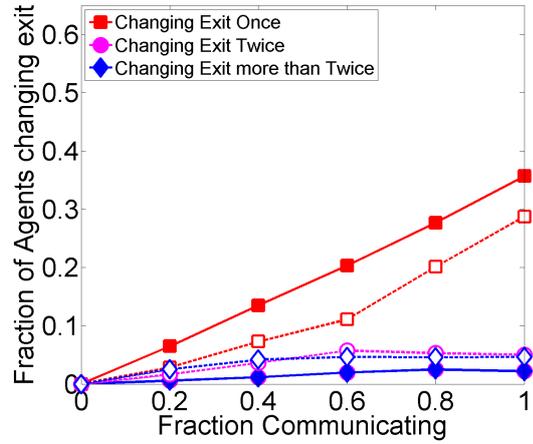
(c) Leaving Time for  $t_I = 30s$



(d) Changing Agents for  $t_I = 30s$

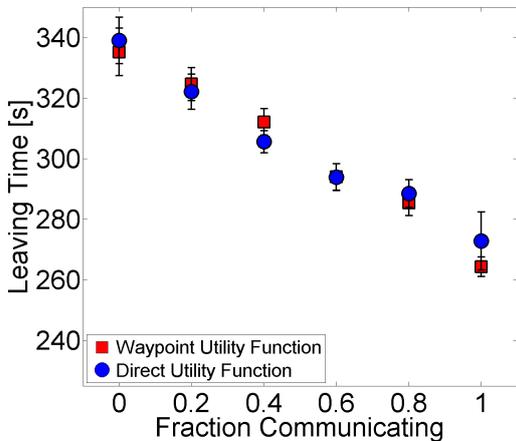


(e) Leaving Time for  $t_I = 60s$

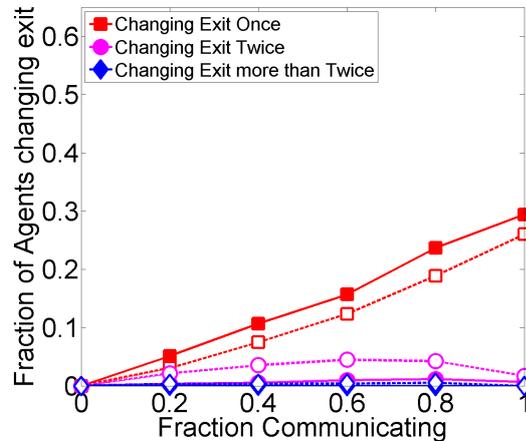


(f) Changing Agents for  $t_I = 60s$

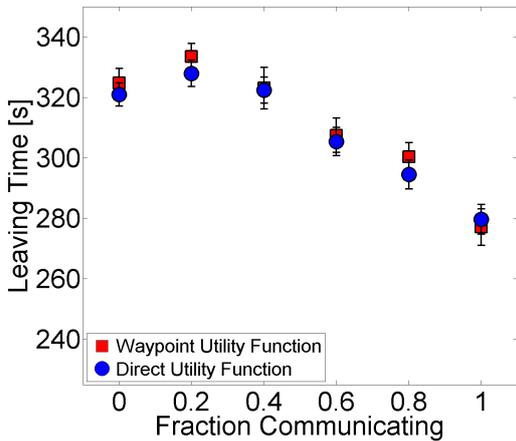
Figure 17



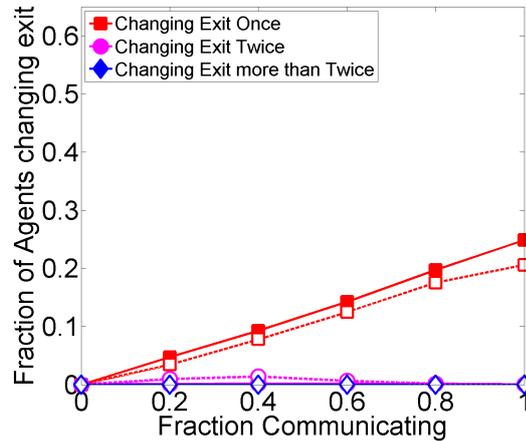
(a) Leaving Time for  $t_I = 90s$



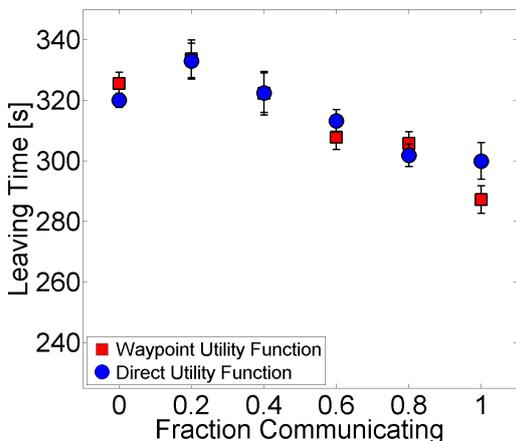
(b) Changing Agents for  $t_I = 90s$



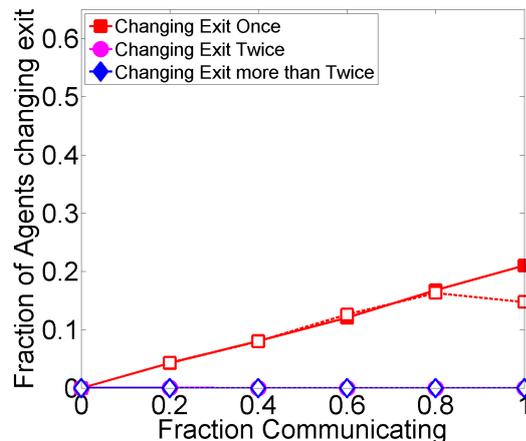
(c) Leaving Time for  $t_I = 120s$



(d) Changing Agents for  $t_I = 120s$



(e) Leaving Time for  $t_I = 150s$



(f) Changing Agents for  $t_I = 150s$

Figure 18